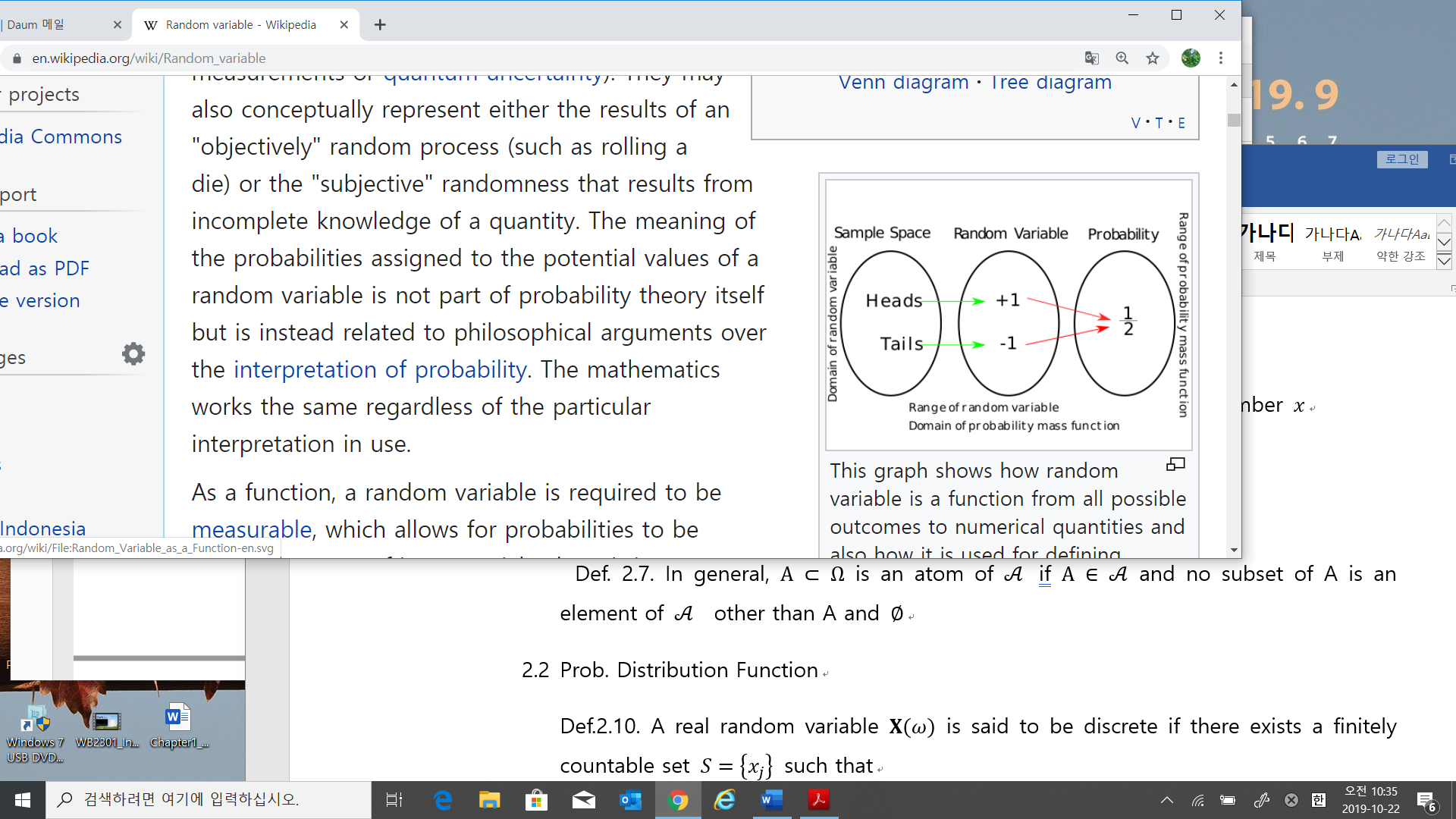
2. Random Variables and Stochastic Process

2.1 Random Variables

Def. 2.1. Given a probability space,, a random variable is a real-(vector-) valued point function which carries a sample point, , into a point in such a way that every sets, , of the form

is an element of the

* The textbook, misspelled. As
* Random variable is a function such that associates to a real number
* Wiki



Def. 2.7. In general, is an atom of if and no subset of A is an element of other than A and

* See the notation ,
* Skip from Ex.2. to the last of 2.1
  1. Prob. Distribution Function(Probability Cumulative Function, CDF)
* Probability Distribution function CDF
* Joint probability distribution function( joint cumulative distribution function)

Let . Then the Joint CDF is

1. The properties of CDF

* is a non-decreasing function, that is,
* is a right-continuous function, s.t.,
* The proofs are in

<http://www.maths.qmul.ac.uk/~bb/MS_Lectures_3and4.pdf>

* The probability of the decomposition of sets

Let so that

Then, using Axiom 2, (1.4), we get

)

)

Hence

* The discontinuity of

If is discontinuous at ,

Ex. 2.4 and Ex 2.9.

Def.2.10. A real random variable is said to be discrete if there exists a finitely countable set such that

* 1. Prob. Density Function
* Suppose , integrable function.

Then is the probability density function(PDF).

Hence is also

Proposition 2.12

* Absolute continuous / continuous 🡪 skip
* Common Distributions Functions for Random Variables

1. Uniform distribution functions
2. Exponential distribution function
3. Gaussian probability distribution.

where

The CDF of Gaussian has not closed form,

For a random vector , the PDF is

where is the covariance matrix, is the determinant of

* 1. Probabilistic Concepts Applied to Random Variables

Def 2.16. Two random variables and are called **independent** if any event of the form is independent of any event of the form where are sets in

* The joint CDF of independent RVs

Let are independent RVs, then

* Marginal CDF : see textbook p.37
  1. Functions of a RV
* PDF of the function of RV

Assume : a continuous function such that

exists

2) and have continuous partial derivatives

Then

Prop. 2.17 Given and and a vector with density function , the n-vector

has the density function

where stands for the absolute value of the determinant of the matrix

Ex. 2.18 🡪 GOOD